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MAIN RELATIONS BETWEEN THE CONSTANTS IN MODELS OF HEATING AND THERMAL DESTRUCTION OF THE SURFACE OF A MATERIAL

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We found the interrelation between the constants that determine the components of heat balance in transition of a material from the solid to liquid and gaseous states under the conditions of thermal destruction of the material's surface. It is shown that the relations between these constants satisfy the "golden proportion." An energy diagram of thermal destruction of the material is suggested. The amount of heat necessary for establishment of the stationary mode of heating and mass entrainment is estimated.

At present, it is accepted the practice to consider four aggregate states of a substance, viz., solid, liquid, gas, and plasma. In transition from one state to another a certain amount of thermal energy is absorbed or released, the properties of the material change sharply, and, as a rule, the material cannot perform its functions. However, determination of the laws governing transition of the material from one aggregate state to another still remains a problem.

Mathematical models constructed to characterize such processes usually involve constants. In this case, equations describing different phenomena can have the same dimensional and dimensionless constants. Recently, much attention has been paid to the Fibonacci number φ. The irrational number $\varphi = 1.618034...$ is the limit to which the ratio of the higher number in the Fibonacci sequence to the neighboring lower number tends. The farther along the sequence, the closer this ratio approaches φ [1]. The numbers 1.618 and 0.618 characterize the "golden proportion" or the "golden section." It has been found that ratios close to the "golden proportion" are widely present in nature. Many examples, starting from such small forms as atomic structures, microcapillaries of the brain, and DNA molecules to such huge ones as planetary orbits and galaxies, show that most natural phenomena can actually be described by the "golden proportion."

The use of the "golden proportion" makes it possible in some cases to predict new materials on the basis of a purely geometric similarity of them [2]. However, this approach does not often allow an explanation of the appearance of one property of the material or another. In our opinion, this is due to the fact that comprehension of the physical essence of the studied phenomena is being lost. All physical processes, including those in transition of the material from one aggregate state to another, are based on the law of conservation of energy. Therefore, consideration of only the geometric characteristics of structures cannot give an adequate understanding of the nature of their origin.

Thus, it is of interest to consider the possible interrelation between different energy constants used in the models of heating and thermal destruction of the material and the Fibonacci number.

As shown in [3], solution of the linear heat-conduction equation for a half-space at the boundary conditions of the first kind is self-similar. At $T_{\text{w}} = \text{const}$ this solution has the form

$$\theta^* = \frac{T^* - T_0}{T_{\text{w}} - T_0} = \operatorname{erfc}\left(\frac{y}{2\sqrt{a\tau}}\right). \tag{1}$$

The depth of the heated layer bounded by the isotherm with dimensionless temperature θ^* obeys the relation

$$y \approx K\sqrt{a\tau}\,, \tag{1$'$}$$

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where K is a coefficient characterizing the velocity of movement of different isotherms. Therefore, K depends on the isotherm under consideration, i.e., on the value of θ^*. Despite the fact that expression (1) involves the parameters y, a, and τ, calculations made within the range of variation of thermal diffusivity by two orders showed that the coefficient K depends only on θ^* and the law of variation of T_w [4]. In this case, the coefficient K reaches the highest value for T_w = const.

Polezhaev and Yurevich [5] have shown that (1) must be used only up to the temperature of destruction (melting) of the material surface T_d, i.e., during time τ_d which under the constant thermal effect q = const is determined by the formula

$$\tau_d = \frac{\pi}{4} \lambda \rho c \frac{(T_d - T_0)^2}{q^2} . \tag{2}$$

From this instant of time thermal destruction of the surface begins, and the heat-conduction equation includes the rate of mass entrainment determined from the corresponding boundary conditions.

However, as it turned out [6, 7], in the nonstationary mode of heating with mass entrainment we can also use an expression similar to (1′), if it is presented in the form

$$\Delta^* = K \sqrt{a} \, (\sqrt{\tau} - \sqrt{\tau_\xi}) , \tag{3}$$

i.e., the position of the isotherm T^* is considered relative to the initial surface. It was found experimentally [7] that for $\theta^* < 0.2$ satisfactory agreement between calculation and experiment is provided by relation (1) and for $\theta^* > 0.2$ it is better to use the equation

$$K = - \frac{1}{K_{T_d}} \theta^* + \frac{K_{T_d}^2}{1 - K_{T_d}} . \tag{4}$$

At θ^* = 1, expression (4) passes over to the third-degree equation

$$2K_{T_d}^3 - K_{T_d}^2 + K_{T_d} - 1 = 0 , \tag{5}$$

whose solution gives a numerical value of the constant of thermal destruction $K_{T_d} \approx 0.739$.

Equation (4) formed the basis of the model of heating and thermal destruction of the material suggested in [8]. However, in [7] only the real root of (5) was considered. At the same time, of no less interest are the complex roots ($x_{1,2} \approx -0.119 \pm i0.814$), since the product of the real root (constant K_{T_d}) by the double square of the modulus $2M^2$ of the root $x_{1,2}$ by the Vieta theorem is just unity:

$$2M^2 = \frac{1}{K_{T_d}} \approx 1.353 . \tag{6}$$

According to [5], upon elapse of a rather large period of the time of heating, the quasistationary mode of heating is established and the temperature profile acquires an exponential form:

$$\theta (y) = \frac{T (y) - T_0}{\bar{T}_w - T_0} = \exp \left(- \frac{\bar{V}_\infty}{a} y \right) . \tag{7}$$

The model of thermal destruction of a material [8] that involves the constant K_{T_d} was improved in [9], where it was shown that the stationary mode of mass entrainment from the surface of the material is established once a layer equal to

$$S(\tau_v) = \frac{d_0}{K_{T_\mathrm{d}}^2} \approx 1.831 d_0 \tag{8}$$

is carried away from the surface. Here d_0 depends virtually only on the thermal conductivity of the material [9, 10].

After entrainment of the layer $S(\tau_v)$ from the material surface, we can assume that the temperature profile for the isotherms with a temperature close to $\overline{T}_\mathrm{w}$ is stationary, since the velocity of these isotherms became equal to the rate of entrainment. The model suggested proposes full coincidence of the current and stationary temperature profiles, rather than gradual approximation, upon transition to the isotherm with a lower temperature [11].

Three conclusions that are of importance for this work follow from this model. First, the stationary mode of heating and mass entrainment virtually for any isotherm that bounds the heated layer is established at the instant of time when the thickness of the entrained layer differs from the depth of isotherm occurrence by no more than 10%; therefore, with an accuracy to the constants d_0 and τ_ξ we can write

$$\frac{\Delta^*(\tau_\delta)}{S(\tau_\delta)} \approx 2\,. \tag{9}$$

Second, for the time of onset of destruction (melting) of the surface we obtained the formula

$$\tau_\mathrm{d} = \frac{K_{T_\mathrm{d}}^6 a}{4\,(K_{T_\mathrm{d}}^2 + 1)^2\, \overline{V}_\infty^2}\,. \tag{10}$$

Third, the equations for calculation of the depth of the heated layer (one following from (7) and another obtained on the basis of the model of [8]) give virtually the same results [11].

Heat-protective materials are often compared by the dimensionless rate of mass entrainment $\overline{G}_\Sigma$ [5], which is equal to the ratio of the rate of material destruction G_Σ to the heat-transfer coefficient $(\alpha/c_p)_0$. In [11] it is shown that two modes of mass entrainment from the surface must be considered. At a dimensionless rate of mass entrainment $\overline{G}_\Sigma \le 0.5$, the rate of destruction is specified by the heat flux (rate of heating) even before the onset of surface destruction and does not depend on the state (solid, liquid, or gas) of the material in which it is carried away from the surface. We can assume that under these conditions the determining role is played by inner processes of heat absorption in the material. On the basis of Eqs. (2) and (10), we suggested a formula for estimation of the dimensionless rate of entrainment, which allows for only the heat content of the material at the temperature of boiling [11]:

$$\overline{G}_\Sigma \approx \frac{\sqrt{\pi}}{1 - K_{T_\mathrm{d}}} \frac{I_\mathrm{e} - I_\mathrm{w}}{H(T_\mathrm{boil})} \approx \frac{I_\mathrm{e} - I_\mathrm{w}}{6.79 H(T_\mathrm{boil})}\,, \tag{11}$$

where the denominator is double the maximum heat of physicochemical conversions on the surface $2(\Delta Q_\mathrm{w})_\mathrm{max}$.

When $\overline{G}_\Sigma > 0.5$, the heat of physicochemical conversions, which includes the heat of evaporation, and the effect of injection rank first in heat balance on the surface of the destructing material. It was proved [12–14] that the parameter of stabilization of mass entrainment that determines the dependence of dimensionless rate of evaporation $\overline{G}_\Sigma$ on the square root of the enthalpy difference on the outer edge of the boundary layer, i.e., on supplied thermal energy, is also equal to $2(\Delta Q_\mathrm{w})_\mathrm{max}$. Probably, the amount of heat, which in a dimensionless form is

$$\frac{2\,(\Delta Q_\mathrm{w})_\mathrm{max}}{H(T_\mathrm{boil})} = \frac{\sqrt{\pi}}{1 - K_{T_\mathrm{d}}} \approx 6.79\,, \tag{12}$$

is the limiting value of thermal energy that must be spent for gasification and entrainment of a solid mass unit with account for blocking of supplied heat by injection of the gasified material into the boundary layer and for radiation from the surface. When $\overline{G}_\Sigma > 0.5$, the equation for calculation of the dimensionless rate of evaporation obtained in [13] has the form

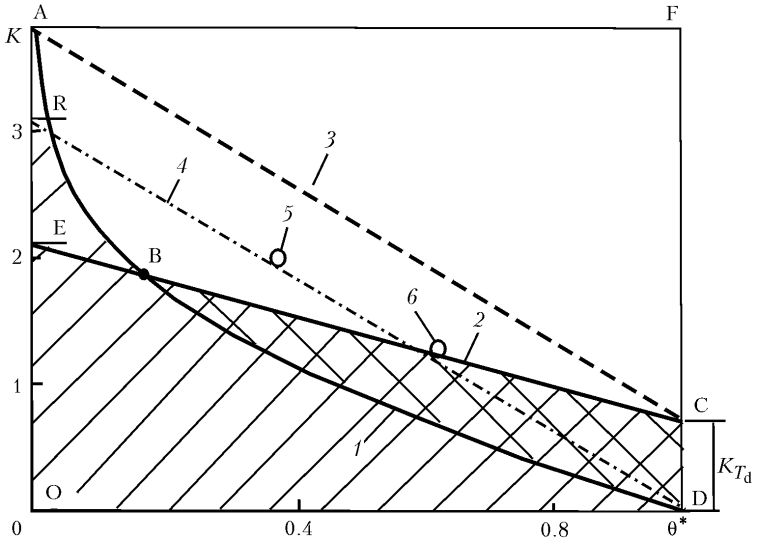

Fig. 1. Energy diagram of thermal destruction of the material. Variation of the coefficient K in material heating to the temperature of surface destruction (melting): 1) T_w = const, calculation by (1); 2) calculation by (4); 3) $\overline{\overline{V}}_\infty \approx 0.32$, calculation by (19); 4, 5, 6) at the moment of establishment of the stationary mode of heating [5, 6) experimental values: 5) $\theta^* = 0.38$; 6) 0.6]; values of the coefficient K at points C, B, E, R, and A — 0.739, 1.831, 2.092, 3.092, and 3.831, respectively.

$$\overline{G}_\mathrm{w} \approx \sqrt{\frac{I_\mathrm{e} - I_\mathrm{w}}{2\,(\Delta Q_\mathrm{w})_\mathrm{max}}} - \frac{H\,(T_\mathrm{boil})}{(\Delta Q_\mathrm{w})_\mathrm{max}} \,. \tag{13}$$

Since for many materials $(\Delta Q_\mathrm{w})_\mathrm{max} \approx \Delta Q_\mathrm{ev}$, the heat of evaporation for most simple substances and SiO_2 was considered [11, 15] and it was proved that with an accuracy up to 7% the relation

$$(\Delta Q_\mathrm{w})_\mathrm{max} \approx \frac{\sqrt{\pi}}{2\,(1 - K_{T_\mathrm{d}})}\, H\,(T_\mathrm{boil}) \approx 3.395 H\,(T_\mathrm{boil}) \tag{14}$$

holds. Then, the heat content of the gasified material $(\Delta Q_\mathrm{w})_\mathrm{max} + H(T_\mathrm{boil})$ in dimensionless form is

$$\frac{(\Delta Q_\mathrm{w})_\mathrm{max} + H\,(T_\mathrm{boil})}{H\,(T_\mathrm{boil})} = \frac{\sqrt{\pi}}{2\,(1 - K_{T_\mathrm{d}})} + 1 \approx 4.395 \,. \tag{15}$$

At present, on the basis of computational-experimental studies conducted in [12–14] and generalization [11], the dependences (11)–(15) can be taken to be strictly proved.

Figure 1 presents the "energy diagram of the material" (EDM) constructed on the basis of Eqs. (1) and (4), which determine the dependence of the coefficient K on dimensionless temperature θ^*: (1) without mass entrainment from the surface of the material at T_w = const and (4) with mass entrainment. Experimental results obtained in the investigations of temperature fields in the specimens of heat-protective materials of different classes, including quartz-glass ceramic [16], confirm the validity of Eq. (4), from which the value of the constant of thermal destruction K_{T_d} is found. Moreover, from (4) a number of proportions that determine the constants (1.831, 2.381, 3.831, etc.) used in Eqs. (8), (10), (11) and the EDM are obtained.

At the same time, Eq. (1), which characterizes the distribution of thermal energy in a solid body until the onset of destruction (melting) of the surface (Fig. 1, curve 1), is obtained for T_w = const, and its use in other laws that take place in heating of a solid body to the temperature of surface melting, can lead to some errors. However, with account for the fact that the time of establishment of constant temperature of the surface is much smaller than

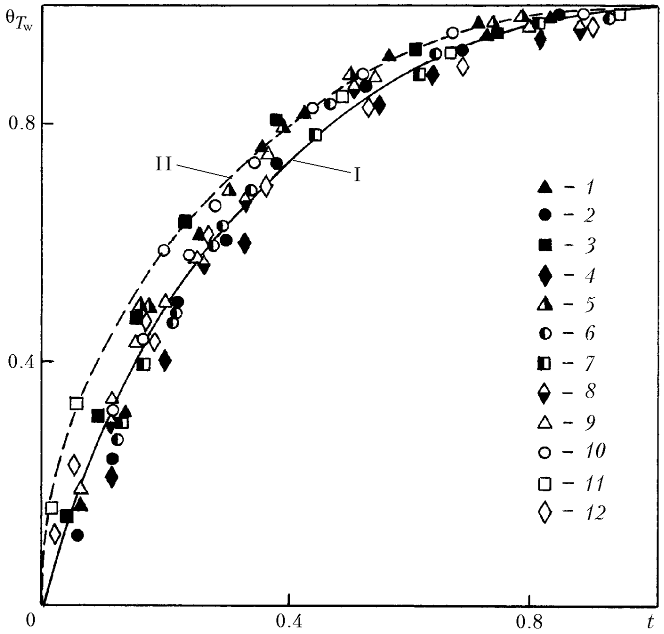

Fig. 2. Dependence of a relative drop of temperatures θ_{T_w} on dimensionless time of heating in the molten layer of quartz-glass-based materials: 1, 5, 9) convective heat flux q_{conv} = 14,700 kW/m^2, P_e = 3.5·10^5 Pa; 2, 6, 10) q_{conv} = 11,500 kW/m^2; 3, 7, 11) q_{conv} = 7650 kW/m^2; 4, 8, 12) q_{conv} = 5850 kW/m^2 (all $P_e \sim 10^5$ Pa); 1–4) doped quartz-glass ceramic; 5–8) pure quartz-glass ceramic; 9–12) quartz glass; I) calculation by (16), II) by (17).

the time of reaching stationary mass entrainment, this error can be neglected. At least, when $\theta^* < 0.2$, even with mass entrainment from the surface, the experimental temperature fields are in good agreement with calculation by (1) [7].

In [17], it was shown that the law of surface temperature variation from the moment when the temperature of melting T_d was reached to the establishment of the constant $\overline{T}_w$ is determined by the constant K_{T_d}. Integration of the dependence that approximates experimental results (Fig. 2)

$$\theta_{T_w} = \frac{T_w - T_d}{\overline{T}_w - T_d} = 1.480t^5 - 5.061t^4 + 7.377t^3 - 6.379t^2 + 3.581t + 9.690 \cdot 10^{-3}\,, \tag{16}$$

gives the value

$$\int_0^1 \theta_{T_w}\, dt = 0.742 \approx K_{T_d}\,.$$

Here $t = (\sqrt{\tau} - \sqrt{\tau_d})/(\sqrt{\tau_T} - \sqrt{\tau_d})$.

It is of interest to note that within the temperature range from T_d to $\overline{T}_w$ the law of surface temperature rise is also described satisfactorily by the equation for a circle of unit radius

$$\theta_{T_w} \approx \sqrt{t\,(2 - t)}\,, \tag{17}$$

and the number π is the coefficient of proportionality between the area on the energy diagram of the material and dimensionless energy ($S_{BCD}\sqrt{\pi} \approx K_{T_d}$).

As a result of accumulation of thermal energy in the surface layer of the material (the area S_{BCD} on the energy diagram of the material), the value of the coefficient K at $\theta^* = 1$ reaches the value of K_{T_d} (Fig. 1, straight line 2). It is seen from Fig. 1 that, in this case, 1 and 2 intersect at point B ($K = 0.831$, $\theta^* = 0.19$). Experimental studies

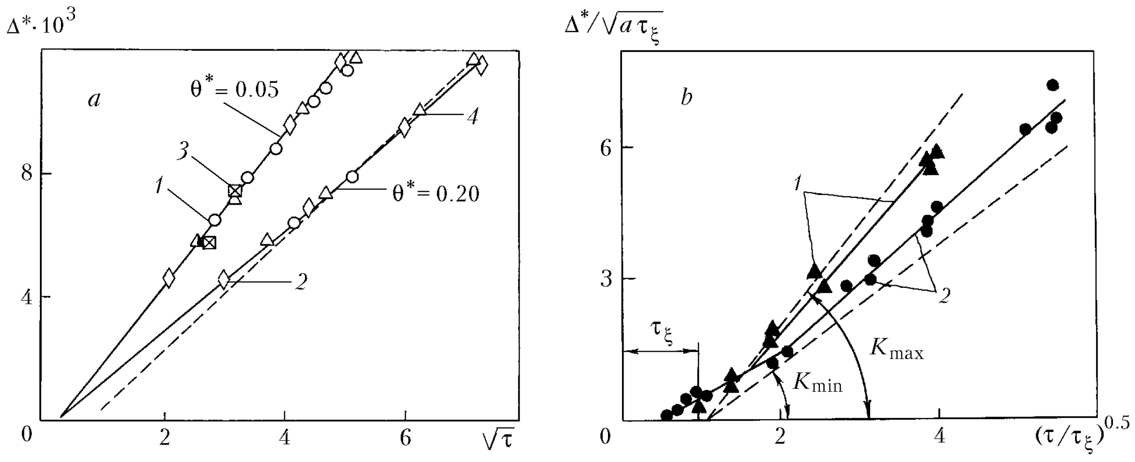

Fig. 3. Dependence of the total thickness of the heated and entrained layers on the time of heating of quartz-glass ceramic: a) $\theta^* \le 0.2$; 1–3) doped quartz-glass ceramic; 4) pure quartz-glass ceramic [1) convective heat flux q_{conv} = 4130; 2, 4) 7260; 3) 9350 KW/m^2]; dots — experiment, curves — processing by the least-squares method; b) $\theta^* \approx 0.6$; 1) $\bar{\bar{V}}_\infty \approx 0.26$; 2) 0.14; dots — experimental data on heating of doped quartz-glass ceramic to T^* = 1800 K; lower dashed line — calculation of K by Eq. (4), upper dashed line — by Eq. (19).

show that this point bounds the region of the temperature field to which the effect of the destructing surface propagates at minimum heat content of the material sufficient for establishment of stationary entrainment of mass from the surface (Fig. 3a). The linear dependence $\Delta^* = f(\sqrt{\tau})$ obtained for the isotherm $\theta^* = 0.2$ in quartz-glass ceramic has a bend, whereas there is no bend for the isotherm $\theta^* = 0.05$. For high-temperature isotherms $\theta^* \approx 0.6$ (Fig. 3b), the dependences $\Delta^* = f(\sqrt{\tau})$ sharply change the angle of inclination on reaching a constant temperature of the surface, i.e., at the instant of termination of heat accumulation in the surface layer of the material.

In [8] it is shown that the law governing mass entrainment in the nonstationary mode is also determined by the constant $K_{T_{\mathrm{d}}}$ and the stationary mode of destruction of the material surface is reached at the instant of time when a layer of the material, whose thickness is determined by (8), will be entrained from the surface. A portion of the thermal energy accumulated during increase in the temperature from T_{d} and $\bar{T}_{\mathrm{w}}$ (area S_{BCD}), which causes the S-shaped character of the temperature profile in the surface layer even of nontransparent material [17], is absorbed by either the layer of the material entrained from the surface [determined according to (8)] or gaseous products of binder decomposition, for example, in filtering through a porous coked layer. Another portion of this energy, which in dimensionless form is $1 - K_{T_{\mathrm{d}}} \approx 0.261$, is accumulated in the stationary heated layer and is spent for inner processes, for example, melting (Fig. 4).

To estimate the heat content of the heated layer we use an expression that follows from (7) and that is obtained in [13]:

$$\frac{H(\bar{T})}{H(T_{\mathrm{w}})} \approx \frac{4}{K^2}\left[1 - \exp\left(-\frac{K^2}{4}\right)\right]. \qquad (18)$$

Since, according to (1), $K \to \infty$ when $\theta^* = 0$, we bound the temperature field by the isotherm $\theta^* = 0.005$ for which $K \approx 3.831$.

The choice of $K \approx 3.831$ is not random since this value is found in experimental studies of the temperature fields in specimens of quartz-glass ceramic. An increase in the density of the supplied heat flux and, consequently, the rate of mass entrainment leads to the fact that for $\bar{G}_\Sigma > 0.5$ the stationary mode is established virtually for all isotherms of the temperature field, when their velocity becomes equal to the velocity of surface movement. With increase in the rate of mass entrainment the path passed by the isotherms in the nonstationary mode of heating increases, i.e.,

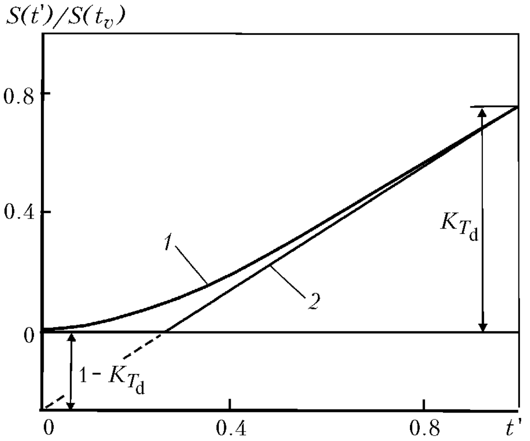

Fig. 4. Dependence of dimensionless linear entrainment $S(t')/S(t_v)$ on the time of heating: 1) without a source of inner gas formation (without cooling of the heated layer); 2) in cooling of the heated layer due to filtration of gaseous products.

the coefficient K depends on the rate of heating. Two important conclusions follow from Fig. 1, where only the bounding dependences $K = f(\theta^*, \bar{\bar{V}}_\infty)$ (curves 2 and 3) which are determined by Eqs. (4) and (19) are given: first, when $\theta^* > 0.2$ the dependences $K = f(\theta^*, \bar{\bar{V}}_\infty)$ are close to linear, and at $\theta^* = 1$ they converge to a common point where $K = K_{T_d}$; second, at a dimensionless rate of entrainment $\bar{\bar{V}}_\infty = \bar{V}_\infty/\sqrt{a\tau_\xi} < 0.05$ (where $\tau_\xi = 1$ sec), the velocity virtually does not influence the value of K and its value for $\theta^* > 0.2$ is determined by (4). Equation (4) also holds at higher rates of surface destruction $\bar{V}_\infty$ if inner sources of gas generation are present in the material and the rate of linear entrainment changes according to dependence 2 (Fig. 4). The approach of $\bar{\bar{V}}_\infty$ to a limiting value of ~0.3 leads to a sharp decrease in the time of existence of linear dependences $\Delta^* = f(\sqrt{\tau})$ (see Fig. 3b). When $\bar{\bar{V}}_\infty \approx 0.3$, the nonstationary period for $\theta^* > 0.2$ can be neglected, and the coefficient K attains maximum values and is described by the expression

$$K = -\left(\frac{1 - K_{T_d} + K_{T_d}^2}{1 - K_{T_d}}\right)\theta^* + \frac{1}{1 - K_{T_d}}, \tag{19}$$

which at $K = K_{T_d}$, in contrast to (4), is generated. The equation of the straight line (19) at $\theta^* = 0$ gives $K \approx 3.831$. This amount of heat in dimensionless form is characterized by the total area on the energy diagram of the material (see Fig. 1). The product $S_{\text{OAFD}}\sqrt{\pi} \approx 6.79$ is equal to the total thermal energy necessary for gasification and entrainment of a mass unit with account for blocking of supplied heat by injection into the boundary layer and radiation from the surface (12).

From Eq. (18) at $K \approx 3.831$ we find that the heat content of the heated layer is ~0.265 of the heat content of the entrained material, which, according to Fig. 4, is of about $(1 - K_{T_d})$. Since the surface temperature can greatly exceed the temperature of material melting (for example, for quartz-glass ceramic by more than 1000 K), the heat content calculated by (18) must also include the heat of melting ΔQ_{melt}. Therefore, estimating the heat content of the solid phase $[H(\bar{T})]_{\text{sol}}$, we must subtract the heat of melting which must be included in the heat content of the material being entrained. The heat of melting of silicon dioxide is ~160 kJ/kg, which is ~0.07 of the heat content of molten silicon dioxide at a temperature of 1996 K [18].

At a dimensionless rate of mass entrainment $\bar{G}_\Sigma > 0.5$ and stagnation enthalpy of the oncoming gas flow higher than 20,000 kJ/kg, destruction of the material surface begins virtually immediately in the stationary mode and almost the whole of the entrained material is gasified. In this case, the total amount of heat necessary for gasification of a material unit is written as follows:

$$2\,(\Delta Q_{\text{w}})_{\max} = \left\{([H(\bar{T})]_{\text{sol}} + \Delta Q_{\text{melt}})_{\text{liq}} + (\Delta Q_{\text{w}})_{\max}\right\}_{\text{gas}} + \Delta Q_{\text{out}}\,. \tag{20}$$

Hence it is seen that the components of heat balance (20) can conventionally be divided into two groups of heat-absorption factors. Although the heat of melting ΔQ_{melt} and heat of physicochemical conversions on the surface $(\Delta Q_{\mathrm{w}})_{\max}$, which includes the heat of evaporation, simultaneously compose a substantial part of the heat balance, they are spent in changing of the aggregate state of the material and, without variation of its form, they cannot be considered as active processes of the heat absorption. At the same time, the heat content of the heated layer $[H(\overline{T})]_{\mathrm{sol}}$, which is ~0.2 of the heat content of the material being entrained from the surface in the liquid or solid state, and the value of ΔQ_{out}, which exceeds this value by an order, must be referred to the factors that directly protect the form of the material from thermal destruction.

The relative value of heat content of the heated layer (0.265) calculated by (18) can be presented as $(1 - K_{T_{\mathrm{d}}}) \approx 0.261$ (Fig. 4), and the heat spent for breaking the bonds in melting (for quartz-glass ceramic 0.07) is almost equal to the difference between the real root of Eq. (5) and the square of the modulus of its roots $x_{1,2}$ (0.062). Then,

$$\frac{[H\,(T)]_{\mathrm{sol}}}{H\,(T_{\mathrm{w}})} = (1 - K_{T_{\mathrm{d}}}) - (K_{T_{\mathrm{d}}} - M^2) \approx 0.261 - 0.062 = 0.199\,. \tag{21}$$

The amount of energy spent for blocking of supplied heat in injection of gaseous products into the boundary layer (plus radiation) is the difference between the total energy of thermal destruction of the material (12) and the heat content of the gasified material (15):

$$\frac{\Delta Q_{\mathrm{out}}}{H\,(T_{\mathrm{boil}})} = \frac{\sqrt{\pi}}{1 - K_{T_{\mathrm{d}}}} - \left(\frac{\sqrt{\pi}}{2\,(1 - K_{T_{\mathrm{d}}})} + 1\right) = \frac{\sqrt{\pi}}{2\,(1 - K_{T_{\mathrm{d}}})} - 1 \approx 2.395\,. \tag{22}$$

Using expressions (12), (15), (21), and (22), we obtain the classical "golden proportion" between thermal energies of material destruction, which indicates that the integer relates to the higher as the higher relates to the smaller:

$$\frac{2\,(\Delta Q_{\mathrm{w}})_{\max}}{[(\Delta Q_{\mathrm{w}})_{\max} + H\,(T_{\mathrm{boil}})] - [H\,(\overline{T})]_{\mathrm{sol}}} = \frac{[(\Delta Q_{\mathrm{w}})_{\max} + H\,(T_{\mathrm{boil}})] - [H\,(\overline{T})]_{\mathrm{sol}}}{\Delta Q_{\mathrm{out}} + [H\,(\overline{T})]_{\mathrm{sol}}} \approx 1.618\,. \tag{23}$$

The ratio of the total energy of thermal destruction of the material to the amount of heat carried away by the products of its destruction is equal to the ratio of this amount of heat to the sum of the thermal energies (injection, radiation, and heat content of the heated layer minus the heat of melting) spent for protection of the form of the material from thermal destruction and satisfies the "golden proportion."

As is found in [11], at a dimensionless rate of entrainment $\overline{G}_{\Sigma} \le 0.5$ the rate of material surface destruction is determined by inner processes of heat absorption and does not depend on the form (solid, liquid, or gas) in which the material is carried away from the surface. The amount of heat that is necessary for establishment of the stationary mode of mass entrainment can be estimated by the energy diagram of the material.

It is seen from Fig. 1 that the minimum amount of heat sufficient for reaching the stationary mode can be proportional to the area $S_{\mathrm{OABCD}} = S_{\mathrm{OEBCD}} + S_{\mathrm{EAB}}$, with the area $S_{\mathrm{OEBCD}} = M^2 + K_{T_{\mathrm{d}}} \approx 1.416$. Graphical integration up to $K_{T_{\mathrm{d}}} \approx 3.831$ gives a value of the area $S_{\mathrm{EAB}} \approx 0.095$. Since the energy diagram of the material characterizes the state of the material after reaching the temperature of melting, the considered thermal energy must, probably, include the latent heat of melting as well. As follows from (23), its value can be taken equal to ~0.062, which after division by $\sqrt{\pi}$ gives a value of 0.035. Then, the minimum amount of heat that must be absorbed by the material for establishment of the stationary mode of mass entrainment in dimensionless form is

$$(1.416 + 0.095 + 0.035)\,\sqrt{\pi} \approx 2.74\,. \tag{24}$$

The amount of heat (~2.74) absorbed by the material in increase of the surface temperature from T_0 to $\overline{T}_{\mathrm{w}}$ provides establishment of the stationary mode of heating and destruction of the material surface. We can expect that redistribution of precisely this amount of thermal energy can, in the long run, provide the exponential temperature profile (7) (Fig. 5a). It should be noted that this amount of heat exceeds by an order the heat content of the stationary

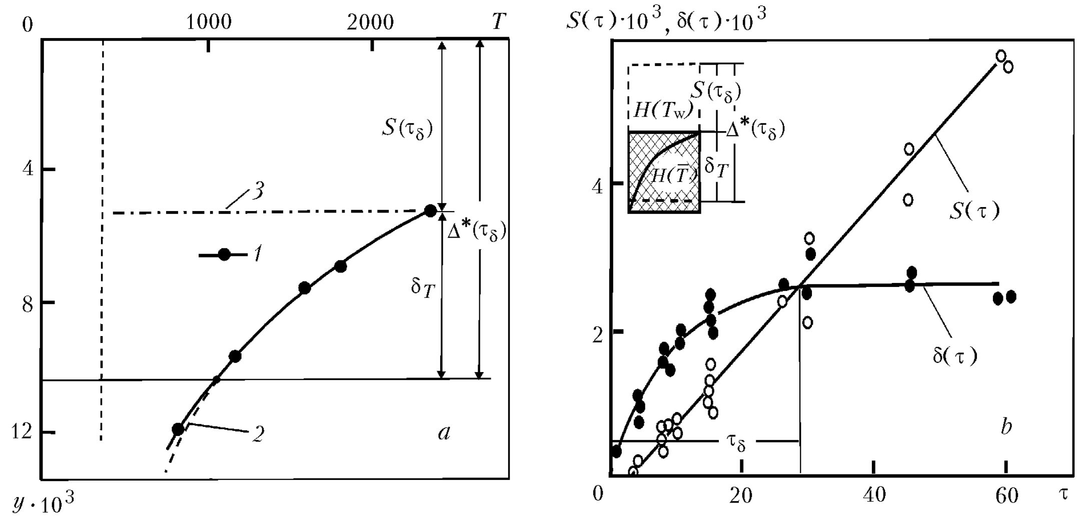

Fig. 5. Determination of the moment of establishment of the stationary mode of heating: a) according to thermocouple measurements, $\theta^* = 0.38$, $\tau = 50$ sec ($q_{\text{conv}} = 7260$ kW/m^2, $T_w = 2390$ K, $\bar{V}_\infty = 0.11\cdot10^{-3}$ m/sec; 1) experiment; 2) stationary temperature profile, calculation by (7); 3) position of the heated surface at the 50th second of heating; b) according to the heated layer to $T^* = 1800$ K in doped quartz-glass ceramic, $\theta^* = 0.6$ ($q_{\text{conv}} = 7650$ kW/m^2, $I_e = 8600$ kJ/kg, $\bar{V}_\infty = 0.1\cdot10^{-3}$ m/sec); dots — experiment. T, K; y, m; $S(\tau)$, m; $\delta(\tau)$, m; τ, sec.

heated layer (~0.2), since in establishing the stationary mode of heating a large part of the thermal energy is absorbed by the material carried away in solid, liquid, and gaseous states.

In order to show that an amount of heat equal to ~2.74 suffices for establishment of the exponential (stationary) temperature profile in the material and to compare its value with the distribution of thermal energy on the energy diagram of the material, it is necessary to present this thermal energy as some area on the energy diagram of the material, allowing for the fact that for such a profile the coefficient K of the surface is zero. However, curves 1 and 2 on the energy diagram of the material describe the nonstationary mode of heating and entrainment of mass and the exponential temperature profile no longer depends on time. This problem can be solved if comparison of the two profiles is made at the instant of equality of the thicknesses of the heated and entrained layers, when the stationary mode of heating is established for the isotherm bounding the heated layer. The coefficient K calculated for this instant of time characterizes the thermal energy of the isotherm bounding the heated layer. To determine the coefficient K at the instant of time t_δ, when the velocity of the isotherm $V_{\theta^*} = \bar{V}_\infty$, we differentiate (3) with respect to τ and obtain

$$K = \sqrt{\frac{4\bar{V}_\infty^2\tau_\delta}{a}}\,. \tag{25}$$

It is seen from Fig. 5a that the calculated curve 2 (exponential profile) at $a = 0.6\cdot10^{-6}$ m^2/sec deviates from the experimental curve starting from a temperature of 1100 K, which corresponds to the isotherm $\theta^* = 0.38$ lying at a distance of $10.3\cdot10^{-3}$ m from the initial surface. At this instant of time (at the 50th second of heating), the thickness of the entrained layer is $5.1\cdot10^{-3}$ m, i.e., the stationary mode of heating for this isotherm is established virtually at the instant of equality of the thicknesses of the heated and entrained layers. One more point that characterizes the moment of establishment of the stationary mode for the isotherm $\theta^* = 0.6$ can be determined by the experimental data obtained on the specimens of doped quartz-glass ceramic for which $a = 0.65\cdot10^{-6}$ m^2/sec (Fig. 5b). Finally, the third point is obtained from the fact that for the exponential profile as well as for the nonstationary profile (1) the coefficient K of the surface isotherm is zero. In Fig. 1 (points 5 and 6) it is shown that experimental values of the coefficient K calculated by (25) are in good agreement with curve 4, i.e., the thermal energy necessary for establishment of the station-

TABLE 1. Ratio of Heat Capacities of Silicon Dioxide in the Gaseous and Liquid States within the Range from Melting Temperature to Boiling Temperature at Atmospheric Pressure [18]

Silicon dioxide in the condensed state, mol. weight 60.0843			Silicon dioxide in the gaseous state, mol. weight 60.0843			c_p^{gas}/c_p^{liq}
T, K	c_p^{liq}, J/(K·mole)	$\Delta H(T)$, kJ/mole	T, K	c_p^{gas}, J/(K·mole)	$\Delta H(T)$, kJ/mole	
1996	75.403	126.836	–	–	–	–
1996	83.500	136.436	–	–	–	–
2000	83.500	136.770	2000	61.308	108.503	0.734
2100	83.500	145.120	2100	61.403	114.638	0.735
2200	83.500	153.470	2200	61.485	120.783	0.736
2300	83.500	161.820	2300	61.557	126.435	0.737
2400	83.500	170.170	2400	61.661	133.094	0.738
2500	83.500	178.520	2500	61.678	139.255	0.739
2600	83.500	186.870	2600	61.728	145.423	0.739
2700	83.500	195.220	2700	61.773	151.604	0.740
2800	83.500	203.570	2800	61.813	157.784	0.740
2900	83.500	211.920	2900	61.849	163.963	0.741
3000	83.500	230.270	3000	61.882	170.153	0.741

ary temperature profile can be presented by the area on the energy diagram of the material $S_{\mathrm{ORD}} = 1 + K_{T_d}^2$, the value of which is also equal to 2.74 ($1.546\sqrt{\pi}$).

The relations between dimensionless energies characterizing thermal destruction of a material that satisfy the "golden proportion" are obtained on the basis of the constant of thermal destruction K_{T_d}. In our opinion, the series of works generalized in [11] substantiate its fundamental importance adequately.

We consider to which natural properties of the material could such a constant be related. By the data of different authors, for example, [19], oxygen and silicon are the most widespread elements in the Earth's crust (O ~ 50%, Si ~ 25–30%). Clearly, materials based on SiO_2 have gained the widest acceptance. In Table 1, values of the ratio of heat capacities of silicon dioxide in the liquid and gaseous states and the heat content of SiO_2 within the range of temperatures from T_d to T_{boil} at atmospheric pressure are presented.

As is seen from the table, the ratio of heat capacities of SiO_2 in the gaseous and liquid states is in rather satisfactory agreement with the constant of thermal destruction $K_{T_d} \approx 0.739$. Using the data of the table, we can determine the heat of melting of silicon dioxide at $T_d = 1996$ K (~9.6 kJ/mole), the value of which in dimensionless form $[\Delta Q_{melt}/H(T_{melt}) = 0.07]$ was used in obtaining relation (23). It should be noted that the ratio of molecular weights of SiO (44.0849) and SiO_2 is also close to the value of K_{T_d} and is equal to 0.734.

NOTATION

a, thermal diffusivity, m^2/sec; c, heat capacity, kJ/(kg·K); c_p^{gas} and c_p^{liq}, heat capacity of the material in the gaseous and liquid states, J/(K·mole); d_0, parameter of nonstationary mass entrainment that specifies displacement of the straight line of entrainment relative to the origin of coordinates, m; $\overline{G}_w$, dimensionless rate of evaporation; G_Σ, rate of mass entrainment, kg/m^2; $\overline{G}_\Sigma$, dimensionless rate of mass entrainment; $H(\overline{T})$, heat content of the heated layer, kJ/kg; $[H(\overline{T})]_{sol}$, heat content of the heated layer without heat of melting, kJ/kg; $H(T_w)$ and $H(T_{boil})$, heat content of the material at the surface temperature and the temperature of boiling, kJ/kg; I_e, stagnation enthalpy, kJ/kg; I_w, enthalpy of the gas at the temperature of the hot surface, kJ/kg; K, coefficient characterizing velocity of isotherm movement; K_{T_d}, constant of thermal destruction; M, modulus of the roots of Eq. (5); P_e, stagnation pressure, Pa; q, heat flux, kW/m^2; q_{conv}, convective heat flux to the cold surface, kW/m^2; $S(\tau)$, $S(\tau_v)$, and $S(\tau_\delta)$, linear entrainment and its values at the moment of reaching stationary values of the rate of entrainment and thickness of the heated layer, m; $S(t')$ and $S(t_v)$, dimensionless linear entrainment and its value at the moment of establishment of the stationary rate of entrainment; $t = (\sqrt{\tau} - \sqrt{\tau_d})/(\sqrt{\tau_T} - \sqrt{\tau_d})$, dimensionless time of heating for the dependence $\theta_{T_w} = f(t)$; $t' = (\tau - \tau_d)/(\tau_v - \tau_d)$, dimensionless time of heating for the dependence $S(t')/S(t_v) = f(t')$; t_v, dimensionless time of reaching the stationary rate of entrainment; T_0, temperature of nonheated material, K; T_w and $\bar{T}_w$, temperature of the heated surface and its stationary value, K; T^*, temperature of the isotherm, K; $\bar{T}$, integral-mean temperature of the heated layer, K; $T(y)$, current value of temperature along the coordinate, K; T_d, temperature of destruction (melting) of the material surface, K; T_{boil}, temperature of boiling, K; V_{θ^*}, velocity of the isotherm, m/sec; $\bar{V}_\infty$, stationary value of the rate of linear entrainment, m/sec; $\bar{\bar{V}}_\infty$, dimensionless stationary rate of linear entrainment; $x_{1,2}$, complex roots of Eq. (5); y, coordinate, m; $(\alpha/c_p)_0$, heat-transfer coefficient, kg/(m^2·sec); $\delta(\tau)$ and δ_T, depth of the heated layer and its stationary value, m; $\Delta H(T)$, heat content of silicon dioxide, kJ/mole; ΔQ_{melt} and ΔQ_{ev}, heat of melting and evaporation, kJ/kg; $(\Delta Q_w)_{max}$, maximum value of heat of physicochemical conversions on the surface, kJ/kg; ΔQ_{out}, heat absorbed due to injection into the boundary layer and radiation from the surface, kJ/kg; Δ^* and $\Delta^*(\tau_\delta)$, total thickness of the entrained and heated layers to the isotherm T^* and its value at the moment of establishment of the stationary mode of heating, m; $\theta^* = (T^* - T_0)/(T_w - T_0)$, dimensionless temperature of the isotherm; θ_{T_w}, dimensionless value of the surface temperature in its variation from T_d to $\bar{T}_w$; $\theta(y)$, current value of dimensionless temperature; φ, Fibonacci number; λ, thermal conductivity, W/(m·K); ρ, density, kg/m^3; τ, current time of heating, sec; τ_T, τ_v, and τ_δ, times of establishment of stationary values of the surface temperature, rate of mass entrainment, and thickness of the heated layer, sec; τ_ξ, section cut off by the linear dependence $\Delta^* = f(\sqrt{\tau})$ on the abscissa axis ($\tau_\xi = 1$ sec); τ_d, time of the onset of destruction (melting) of the surface, sec. Indices: 0, nonheated material, impermeable surface; e, outer edge of the boundary layer; p, pressure; T, temperature; v, velocity; w, wall conditions; Σ, total; δ, heated layer; out, outer; gas, gaseous; liq, liquid; ev, evaporation; conv, convective; boil, boiling; melt, melting; d, destruction; sol, solid; ∞, infinity.

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